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14. ABSTRACT

This paper describes the benefits of transmission subspace tracking for multiple input multiple output communications systems, and applies the concepts of previous work on adaptive transmit antenna algorithms to this problem. A specific stochastic gradient technique of subspace or "multi-mode" tracking of the independent modes of the MIMO transfer function and the application to space time coding is considered. The technique provides gains by tracking the active modes when there are more transmit than receive antennas. With the proposed algorithm, the receiver generates binary feedback selecting preferred perturbed weights, which gives the transmitter a gradient estimate useful for subspace tracking. The capability resulting from this approach is shown by simulation.

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Transmission Subspace Tracking for MIMO Communications Systems

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Abstract - **This paper describes the benefits of transmission subspace tracking for multiple input multiple output communications systems, and applies the concepts of previous work on adaptive transmit antenna algorithms to this problem. A specific stochastic gradient technique of subspace or "multi-mode" tracking of the independent modes of the MIMO transfer function and the application to space time coding is considered. The technique provides gains by tracking the active modes when there are more transmit than receive antennas. With the proposed algorithm, the receiver generates binary feedback selecting preferred perturbed weights, which gives the transmitter a gradient estimate useful for subspace tracking. The capacity resulting from this approach is shown by simulation.**

I. INTRODUCTION

In recent years there have been advances in multiple-input multiple-output (MIMO) communications systems, specifically the concept of "space time coding" (STC) for wireless communications [5][6][11]. The STC algorithms use the independent modes of the MIMO transfer function to effectively attain multiple parallel transmission pipes, giving an increase in the effective available transmission bandwidth and allowing for greater transmission bit rates. The optimal transmission algorithm uses a "water filling" power/rate allocation strategy [12] into these non-zero modes. In frequency division duplex systems only the receiving unit can measure the channel transfer function, and hence feedback is required to provide this information to the transmitting unit and the complexity of water filling may be prohibitive. This paper describes an algorithm for tracking the principal non-zero modes of the transfer function utilizing feedback, without the additional complexity of doing power/rate allocation into those modes.

Tracking of the principal modes provides benefit when some of the available transmission subspaces have null or near null response and hence deliver no power to the receiver. This condition will arise if the channel response is ill-conditioned due to a poor scattering environment, as can occur even with fading independent across all antennas [7], or if the number of receive antennas is less than the number of transmit antennas due to cost or size constraints. The specific algorithm proposed is a subspace tracking variation on the algorithm previously analyzed in [1][2], which is a single receive antenna subset of the algorithm described in the present paper.

There is a substantial body of literature on the subject of signal processing approaches for transmit adaptation in multiple-input single-output (MISO) environments and for subspace tracking in the context of receive systems, examples being [2][9] and [10][13][14] respectively. The focus of this paper is a signal processing approach to tracking the transfer function principal subspace, without consideration of the coding techniques applied once those modes are determined. Any of a number of standard coding techniques can be applied, and the MIMO transfer function modes can be utilized the same as any time/frequency modes are used for traditional coding.

The algorithm uses feedback to provide the transmitter with a coarse estimate of the gradient of the power delivered to the receiver with respect to the transmission weight vectors. Gaussian perturbation probing is applied to the weight vectors of the pilots of each space time coded transmission stream such that the receiver can select a preferred perturbed weight vector set and simultaneously extract a channel estimate for each code stream. Feedback is generated to select the preferred weight set, which provides the transmitting unit with the coarse gradient estimate used to update the weights. Gram-Schmidt orthonormalization is performed to maintain the orthonormality of the applied weight set. The first order behavior of this approach is very similar to that of [14].

Simulations with 2 receive and 4 or 8 transmit antennas in a fading environment show the effectiveness of this tracking algorithm in attaining the potential gain over blind MIMO transmission, and capacities for optimal water filling and optimal subspace tracking are generated for comparison.

II. MIMO CAPACITY CONSIDERATIONS

A. MIMO Capacity for Blind Transmission

The MIMO system will have N_T transmission antennas and N_R receive antennas, where it will be assumed that $N_T > N_R$. We consider first blind transmission, where the channel input is a $N_T\times 1$ vector $\mathbf{t}$, the channel output is the $N_R\times 1$ vector $\mathbf{r}$, the channel gain is a $N_R\times N_T$ matrix $\mathbf{H}$, and the noise is a zero mean $N_R\times 1$ complex gaussian vector $\mathbf{n}$ with autocorrelation $2\sigma^2\mathbf{I}$. The total transmitted power is constrained to $P^{(T)}=E(\mathbf{t}^H\mathbf{t})$.

$$\mathbf{r}=\mathbf{H}\mathbf{t}+\mathbf{n} \tag{1}$$

A coded transmission vector $\mathbf{t}$ will be uncorrelated and the total transmission power is $P^{(T)}$, so that

$$E\left(\mathbf{t}\mathbf{t}^H\right)=\frac{P^{(T)}}{N_T}\mathbf{I} \tag{2}$$

The channel capacity for this system is [5]

$$C_{blind}=\log_2\left(\left|\mathbf{I}+\frac{P^{(T)}}{2N_T\sigma^2}\mathbf{H}^H\mathbf{H}\right|\right) \tag{3}$$

where $|.|$ and $(.)^H$ denote the matrix determinant and conjugate transpose respectively.

B. MIMO Capacity for Subspace Tracking Transmission

The proof of (3) through the singular value decomposition (SVD) of $\mathbf{H}$ [12] is informative with regard to the physical nature of the capacity through the existence of parallel channels (eigenmodes) and the benefits of tracking those modes.

The SVD of $\mathbf{H}$ with magnitude sorted singular values $\lambda_k^{1/2}$ and unitary matrices $\mathbf{U}$ and $\mathbf{V}$ is

$$\underset{N_R\times N_T}{\mathbf{H}}=\underset{N_R\times N_R}{\mathbf{U}}\;\underset{N_R\times N_T}{\mathbf{\Lambda}^{1/2}}\;\underset{N_T\times N_T}{\mathbf{V}}^H \tag{4}$$

Then the channel formulation (1) can be cast into an equivalent form with $\tilde{\mathbf{r}}$, $\tilde{\mathbf{t}}$ and $\tilde{\mathbf{n}}$.

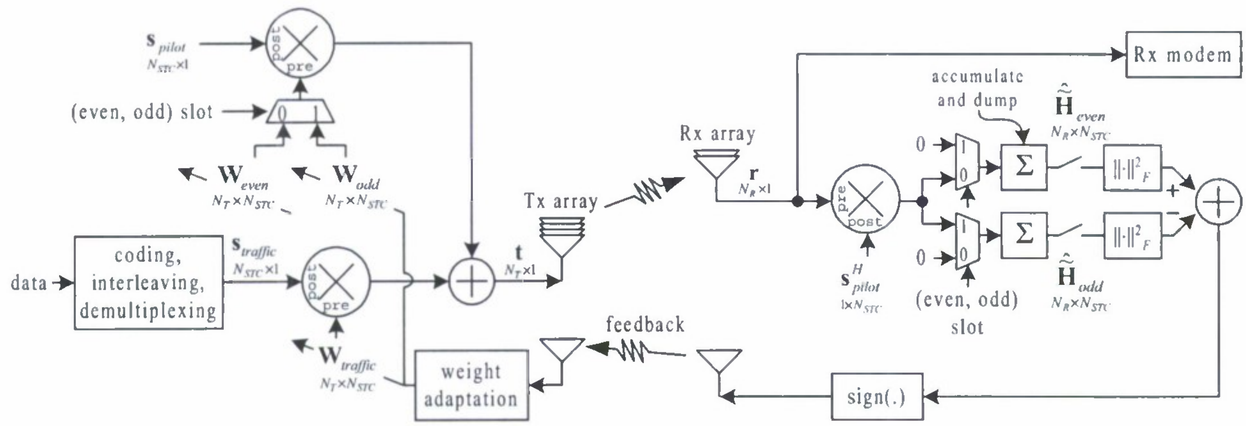

Figure 1: Diagram of the system with the gradient algorithm

$$\begin{aligned}\tilde{\mathbf{r}} &= \mathbf{U}^H\mathbf{r} \\ &= \Lambda^{1/2}\mathbf{V}^H\mathbf{t} + \mathbf{U}^H\mathbf{n} \\ &= \Lambda^{1/2}\tilde{\mathbf{t}} + \tilde{\mathbf{n}}\end{aligned} \tag{5}$$

Since $\mathbf{V}$ and $\mathbf{U}$ are an unitary matrices, $\tilde{\mathbf{t}}$ and $\tilde{\mathbf{n}}$ have the same equidiagonal second moments as their source vectors $\mathbf{t}$ and $\mathbf{n}$, and the independent channels from $\tilde{\mathbf{t}}$ to $\tilde{\mathbf{r}}$ provide the capacity "multiplexing gain".

$$C_{blind} = \sum_{k=0}^{\min(N_R,N_T)-1} \log_2\left(1 + \frac{P^{(T)}}{2N_T\sigma^2}\lambda_k\right) \tag{6}$$

Equation (6) directly yields (3). From the perspective of the transmitter, these independent channels are the eigenmodes of the $N_T \times N_T$ squared gain matrix $\mathbf{R}$.

$$\mathbf{R} \equiv \mathbf{H}^H\mathbf{H} = \mathbf{V}\Lambda\mathbf{V}^H \tag{7}$$

An "informed" transmitter, which knows the realization of $\mathbf{H}$, can attain better performance than that described above, optimizing power and rate allocation to the eigenmodes using "water filling" [4][12]. This requires accurate knowledge of the matrix $\mathbf{H}$. A simpler approach becomes apparent from considering (5) and (6) when $N_T > N_R$. Since the rank of $\mathbf{R}$ is the minimum of N_R and N_T, there are null spaces in $\mathbf{R}$, and the blind transmitter is transmitting power into those null spaces; none of this power reaches the receiver. Transmitting only into the non-zero eigenmodes, the autocorrelation of the modulation vector $\mathbf{t}$ becomes

$$E(\mathbf{t}\mathbf{t}^H) = \frac{P^{(T)}}{\sum_{k=0}^{N_T-1}(\lambda_{N_T-1} > 0)}\mathbf{V}\begin{bmatrix}(\lambda_0 > 0) & & \\ & \ddots & \\ & & (\lambda_{N_T-1} > 0)\end{bmatrix}\mathbf{V}^H \tag{8}$$

where $(\lambda_k > 0)$ represents the numerical value of the boolean: 0 if false, 1 if true.

Applying (5) and (6) with this modified modulation scheme with $N_T > N_R$, we find

$$C_{\text{mode tracking}} = \log_2\left|\mathbf{I} + \frac{P^{(T)}}{2N_R\sigma^2}\mathbf{H}^H\mathbf{H}\right| \tag{9}$$

Hence, a capacity improvement of $10\cdot\log_{10}(N_T/N_R)$ dB arises from this multi-mode tracking without full water filling. The object of the algorithm presented in this paper is to track those non-zero modes, i.e. the subspace orthogonal to the nullspace of $\mathbf{R}$.

III. Algorithm Description

A. Algorithm Operation

The algorithm goal is to use feedback from the receiver to assist the transmitter in tracking the dominant modes of the channel matrix and to accomplish transmission according to (8). This is accomplished first by defining a code stream vector $\mathbf{s}$ of dimension $N_{STC}\times 1$. N_{STC} is less than or equal to the overall rank of $\mathbf{H}$, as determined by some combination of capacity requirements, number of transmit and receive antennas, and perhaps measurement by the receiver determining the channel's usable rank, as this might be less than $\min(N_T,N_R)$ in many environments. The code stream $\mathbf{s}$ is then mapped to the transmission antennas by a set of N_{STC} transmission weight vectors contained in the $N_T \times N_{STC}$ matrix $\mathbf{W}$.

The desired matrix $\mathbf{W}$ is clearly defined from (8) as a matrix with a column span occupying the subspace given by the principal vectors of $\mathbf{V}$, with $\mathbf{W}^H\mathbf{W}=\mathbf{I}$. Mechanisms previously proposed for tracking single mode transmit weights (i.e. MISO systems) can be extended to the multi-mode tracking case, such as multiple vector code book selection extrapolated from [8]. The flexibility and performance of the gradient algorithm considered in [1][2] make it appealing as a source for generalization to multi-mode tracking, and this is the approach considered here.

The system incorporates pilot sequence transmissions with each parallel coded traffic transmission, shown in Figure 1. The weight set applied to the pilot is even and odd time multiplexed between perturbed matrices so that the receiver can generate a channel estimate for demodulation and at the same time generate a single feedback bit selecting the preferred perturbed pilot weight matrix. Considering sampling at the modulation symbol (or chip) rate with Nyquist filtering the transmission $\mathbf{t}$ is

$$\begin{aligned}\underset{N_T\times 1}{\mathbf{t}}(i) = &\sqrt{P_{traffic}^{(T)}}\,\underset{N_T\times N_{STC}}{\mathbf{W}_{traffic}}\left(\left\lfloor \frac{i}{M_{slot}M_{meas}}\right\rfloor\right)\underset{N_{STC}\times 1}{\mathbf{s}_{traffic}}(i) \\ &+ \sqrt{P_{pilot}^{(T)}}\,\underset{N_T\times N_{STC}}{\mathbf{W}_{pilot}}\left(\left\lfloor \frac{i}{M_{slot}}\right\rfloor\right)\underset{N_{STC}\times 1}{\mathbf{s}_{pilot}}(i)\end{aligned} \tag{10}$$

where M_{slot} is the number of modulation samples per perturbation slot and M_{meas} is the number of perturbation slots per measurement/feedback interval. With a perturbation parameter β_1 and a perturbation matrix $\mathbf{P}$ the even/odd time multiplexed weights are

$$k \equiv \left\lfloor \frac{i}{M_{slot} M_{meas}} \right\rfloor \tag{11}$$

$$\mathbf{W}_{pilot}\left(\left\lfloor \frac{i}{M_{slot}} \right\rfloor\right) = \begin{cases} \mathbf{W}_{even}(k) = \mathbf{W}_{base}(k) + \beta_1 \mathbf{P}(k) & i/M_{slot} \text{ even} \\ \mathbf{W}_{odd}(k) = \mathbf{W}_{base}(k) - \beta_1 \mathbf{P}(k) & i/M_{slot} \text{ odd} \end{cases} \tag{12}$$

$$\mathbf{W}_{traffic}(k) = \frac{\mathbf{W}_{even}(k) + \mathbf{W}_{odd}(k)}{2} \tag{13}$$

With frequency flat fading, perfect timing recovery and matched Nyquist receive filtering with noise $\mathbf{n}(i)$ the received signal is

$$\begin{aligned} \mathbf{r}(i) &= \mathbf{Ht}(i) + \mathbf{n}(i) \\ &= \sqrt{P_{traffic}^{(T)}}\,\tilde{\mathbf{H}}_{traffic}(k)\mathbf{s}_{traffic}(i) + \sqrt{P_{pilot}^{(T)}}\,\tilde{\mathbf{H}}_{even/odd}(k)\mathbf{s}_{pilot}(i) + \mathbf{n}(i) \end{aligned} \tag{14}$$

where $\tilde{\mathbf{H}}$ denotes the composite weight matrix and channel as seen by the receiver. As suggested in [1], the receiver can simply extract an estimate of the channel $\tilde{\mathbf{H}}_{traffic}(k)$ from estimates of $\tilde{\mathbf{H}}_{even}(k)$ and $\tilde{\mathbf{H}}_{odd}(k)$. In addition, once per measurement period, indexed k, the receiver generates feedback selecting which of the two weight matrices provided the larger total received power using the Frobenius norm.

$$d(k) = sign\left(\left\| \hat{\tilde{\mathbf{H}}}_{even}(k) \right\|_F^2 - \left\| \hat{\tilde{\mathbf{H}}}_{odd}(k) \right\|_F^2 \right) \tag{15}$$

Upon receiving this feedback, the transmitter can update the weight matrix to the orthonormalized version of the receiver preferred matrix by setting $\beta_2=\beta_1$ in (16), or using an update size β_2 distinct from the measurement perturbation β_1. Including these two parameters allows the measurement and update to be optimized separately. A Gram-Schmidt orthogonalization is then used, denoted $Q(.)$ from the QR decomposition. The transmitter update is then

$$\mathbf{W}_{base}(k+1) = Q\left(\mathbf{W}_{base}(k) + \beta_2 \cdot d(k) \cdot \mathbf{P}(k)\right) \tag{16}$$

B. Convergence Performance

The inverse cost function of the adaptive system is the total received power, denoted J.

$$J(k) \equiv \left\| \tilde{\mathbf{H}}(k) \right\|_F^2 = \mathrm{tr}\left(\mathbf{W}^H(k)\mathbf{R}\mathbf{W}(k)\right) \tag{17}$$

It is shown in [14] that the maximization of the quantity (17) accomplishes the desired subspace tracking, and the gradient of J is

$$\nabla_{\mathbf{W}} J(k) = 2\mathbf{R}\mathbf{W}(k) \tag{18}$$

Extending a result from [3] (Append. A), if $\mathbf{P}$ is comprised of i.i.d. random complex gaussians with variance twice unity, the expected value of the weight change prior to orthonormalization is the scaled normalized gradient of J with respect to $\mathbf{W}$. Hence, assuming that β_1 is small enough for 1st order approximation of the result, neglecting estimation error in the receiver, and assuming reliable feedback, the weight matrix update is

$$\begin{aligned} \mathbf{W}(k+1) &= Q\left(\mathbf{W}'(k+1)\right) \\ &= Q\left(\mathbf{W}(k) + \beta_2 \sqrt{\frac{2}{\pi}} \frac{\mathbf{R}\mathbf{W}(k)}{\left\|\mathbf{R}\mathbf{W}(k)\right\|_F} + \beta_2 \mathbf{E}(k) \right) \end{aligned} \tag{19}$$

where the prime represents the update just prior to orthonormalization and $\mathbf{E}$ is a zero mean error matrix. $\mathbf{E}$ has some autocorrelation, due to the normalized gradient, which is extracted from $\pm\beta_2\mathbf{P}$ to leave $\mathbf{E}$. The eigenmodal representation in the update prior to orthonormalization is then a diagonal modification plus noise from $\mathbf{E}$.

$$\mathbf{V}^H\mathbf{W}'(k+1) = \left(\mathbf{I} + \beta_2 \sqrt{\frac{2}{\pi}} \frac{\Lambda}{\left\|\Lambda \mathbf{V}^H \mathbf{W}\right\|_F} \right) \mathbf{V}^H \mathbf{W}(k) + \beta_2 \mathbf{V}^H \mathbf{E}(k) \tag{20}$$

As discussed in [14], a gradient update with the form of (20) will cause the weight matrix to converge in expected value to the principal subspaces of $\mathbf{R}$. Considering that the direction of the first column of $\mathbf{W}'$ is unchanged by the Gram-Schmidt procedure, it is clear from the principle of matrix power iteration that this column of $\mathbf{W}$ will track towards the principal eigenvector of $\mathbf{R}$, the second column of $\mathbf{W}$ will track towards the second principal eigenvector, etc.

IV. Simulation Results

A. Simulation Environment

The algorithm is simulated as described above with $N_{STC}=N_R=2$ and both $N_T=4$ and $N_T=8$. In all cases $\beta_1=0.005$, small enough to ensure 1st order gradient extraction without 2nd order effects in the receiver measurement. Channel estimation at the receiver was considered to be perfect for purposes of generating the feedback and capacity calculations, a not unreasonable assumption when the data rate is much larger than the channel fading rate, which allows for a relatively small pilot transmission power. The feedback is received without errors, and β_2 was varied to find its best value. The channel model is independent Raleigh flat fading with time correlation given by Jakes model and a doppler frequency of 5Hz.

Two cost metrics were evaluated through the simulation:

$$J_0 = \frac{E(J(k))}{E(J_{opt}(k))} \tag{21}$$

$$J_1 = E\left(\frac{J(k)}{J_{opt}(k)}\right) \tag{22}$$

where $J(k)$ is the time varying value of (17) and $J_{opt}(k)$ is the time varying value of (17) for perfect subspace tracking. In addition, mean capacity values in units of bits/second/Hertz were evaluated according to

$$\bar{C} = E\left(\log_2 \left(\left| \mathbf{I} + \frac{P^{(T)}}{2N_{STC}\sigma^2} \mathbf{W}^H \mathbf{H}^H \mathbf{H} \mathbf{W} \right| \right) \right) \tag{23}$$

These capacities were evaluated for several conditions for MIMO channels with both 8 and 4 transmit antennas:

SISO AWGN:	SISO system, $N_T=N_R=1$, with no fading (AWGN channel, baseline reference)
SISO:	$N_T=N_R=1$
SIMO:	$N_T=1$, $N_R=2$
MIMO WF:	perfect water filling
MIMO OST:	optimal subspace tracking (error free tracking of the transmission eigenmodes)
MIMO Bld:	blind transmission into all transmit subspaces
MIMO GA:	gradient algorithm subspace tracking

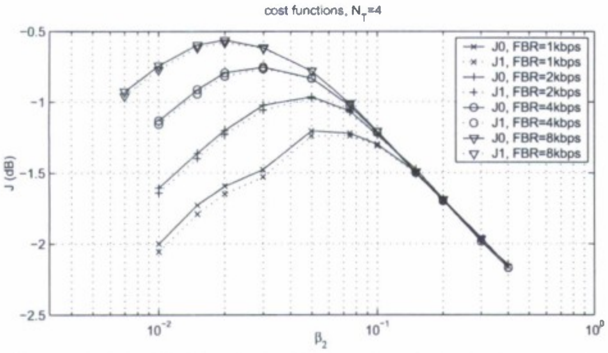

Figure 2: Simulated gradient subspace tracking cost functions J_0 and J_1 vs adaptation β_2; various feedback rates "FBR"; N_T=4, N_R=2

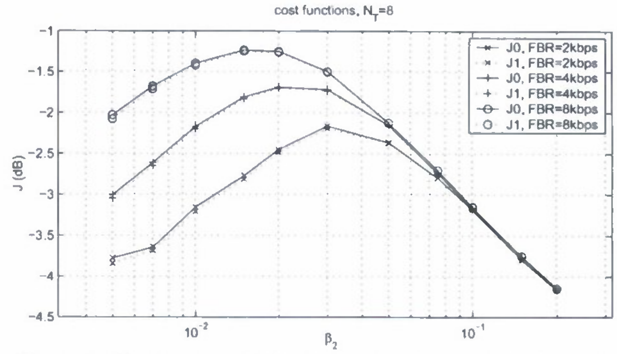

Figure 5: Simulated gradient subspace tracking cost functions J_0 and J_1 vs adaptation β_2; various feedback rates "FBR", N_T=8, N_R=2

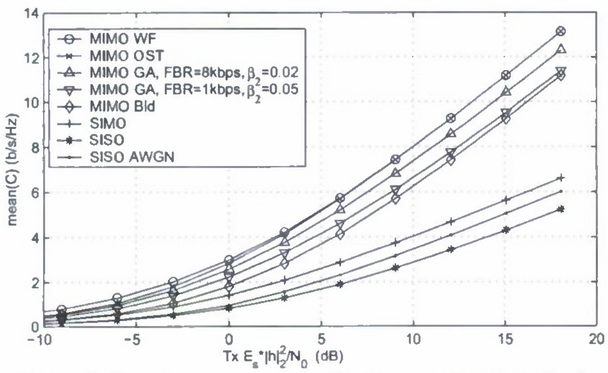

Figure 3: Capacity vs. energy per Nyquist sample, N_T=4, N_R=2. See key in section IV-A. .

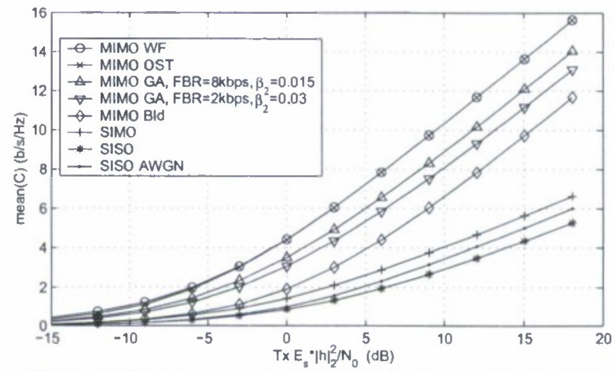

Figure 6: Capacity vs. energy per Nyquist sample time, N_T=8, N_R=2. See key in section IV-A. .

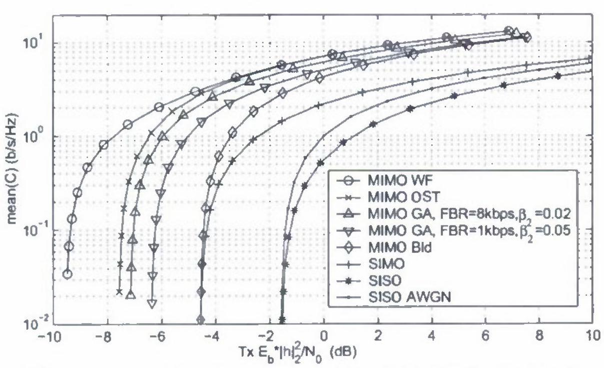

Figure 4: Capacity vs. energy per bit, N_T=4, N_R=2. See key in section IV-A. .

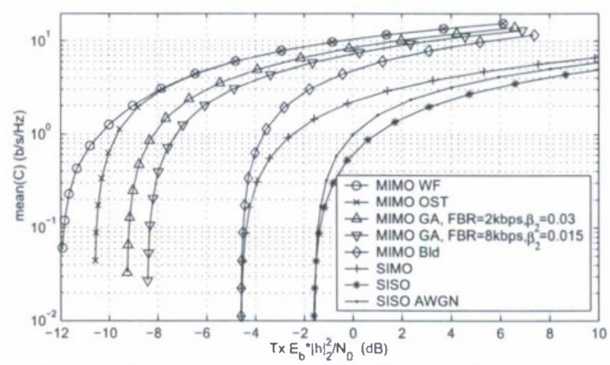

Figure 7: Capacity vs. energy per bit, N_T=8, N_R=2. See key in section IV-A. .

B. Discussion

The key capacity results are given in Figure 3, Figure 4, Figure 6, and Figure 7, showing the gains available from the use of space time coding in the MIMO environment and the tradeoffs associated with the selection of the strategy. The x-axis energy per bit or energy per Nyquist symbol are shown for a total transmit power and mean single channel gain $|h|_2$, so that the benefit of directing the transmission power toward the receiver is visible.

$$|h|_2 = \sqrt{E|h_{i,j}|^2} \quad (24)$$

Consider the capacity plots as the system approaches infinitesimal data rate in "power limited" operation in Figure 4 and Figure 7. The reference AWGN SISO system asymptote is the well known -1.6dB E_b/N_0 limit. The 1×1 SISO system in fading approaches the same limit, as the infinitesimal data rate gives infinite time diversity to each transmission symbol; for higher data rates we see the performance degradation of the SISO system due to the fading channel. The required E_b/N_0 for both 1×2 SIMO and N_T×2 blind MIMO systems approach the same asymptote for infinitesimal data rate: -4.6dB, representing a 3dB performance enhancement relative to the AWGN baseline due to the 2 receive antennas. It is interesting to note that in this limit the "multiplexing gain" afforded to the MIMO system by the multiple transmit antennas does not afford any performance improvement with blind transmission; one might as well transmit with only 1 antenna. The optimal N_T×2 subspace tracking MIMO systems provide an additional performance gain of $10 \cdot \log_{10}(N_T/N_R)$ dB relative to the 1×2 SIMO or N_T×2 blind MIMO systems, for gains of 6dB and 9dB over the AWGN reference for N_T=4 and N_T=8 respectively. Finally, the perfect water filling system can squeeze out 1.95dB for 4×2 MIMO or 1.41dB for 8×2 MIMO beyond what the optimal subspace tracking implementation gives.

In Figure 3 and Figure 6 the capacities as a function of transmission power are shown. In the "bandwidth limited" condition the performance of all of the MIMO algorithms provide 2bit/3dB of capacity improvement as the power is increased, as is expected for the two mode MIMO environment, while the SIMO and SISO algorithms show 1bit/3dB with only one transmission mode. In this region the optimal subspace tracking algorithm provides the same performance as perfect water filling, and continues to give a $10 \cdot \log_{10}(N_T/N_R)$ dB performance gain over blind MIMO transmission.

Together, the capacity plots show the gains available by subspace tracking. In order to obtain substantial gains from a MIMO system operating in the power limited portion of the capacity curve where $N_R < N_T$ some form of transmission adaptation is clearly required, as the blind space time coding transmission does not offer substantial gain in this operating region. Since many practical systems will be interference limited, and additional power transmitted for each information bit transferred is interference to other users, there is strong motivation for operating in or near the power limited rather than the bandwidth limited portion of the capacity curve. Hence, we see the motivation for adopting some form of transmission adaptation scheme, with subspace tracking forming perhaps the simplest general class of appropriate adaptation.

The simulation results of Figure 2 and Figure 5 show the performance of the proposed gradient algorithm for various parameters. There is a strong dependency on both the selection of β_2 and of the feedback rate. Increasing the feedback rate allows for faster adaptation with a smaller β_2 and significantly improves the performance of the gradient subspace tracking algorithm. For the simulated fading rate of 5Hz, it is clear that the receive unit to transmit unit sign feedback bit rate must be at least on the order of 1kHz, and approaching 10kHz provides near optimum performance for the 4×2 MIMO case. The realized mean capacity for the gradient algorithm with the best values of β_2 for are also shown in Figure 3, Figure 4, Figure 6, and Figure 7, with feedback rates of 8kbps and 2kbps (N_T=8) and 1kbps (N_T=4). These show that the loss captured in the performance metrics J_0 and J_1 are reasonable approximations of the actual loss of capacity relative to perfect subspace tracking.

Obtaining reasonably high feedback bit rates, on the order of 1000× the fading rate, appears to be a requirement for good operation of the gradient feedback algorithm. This is not likely to be a large cost in overall bandwidth resource, since the data rates on the forward link would likely be far larger, but it also implies a small channel estimation time for each decision, which is important when imperfect channel estimation problems are considered. While the algorithm performs largely as expected and desired, the practical realization of a system of the type proposed in this paper requires further research. The gains which are possible from the algorithm are clearly substantial.

V. Conclusion

The desirability of subspace tracking algorithms for low rank MIMO systems has been identified and verified through analysis and simulation. For systems with more transmit than receive antennas the gain available from multi-mode tracking is $10 \cdot \log_{10}(N_T/N_R)$ dB over the application of blind, non-adaptive, space time coding techniques. It has been shown that blind space time coding techniques give very little gain if the system is operating in the "power limited" region of the capacity curve, which is a desirable operation region from the standpoint of minimizing interference. Finally, a specific gradient algorithm incorporating feedback from the receiver to track transmission weights to the principal channel transfer modes has been presented and shown to be capable of providing performance near that of perfect subspace tracking.

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